

Experimental nuclear masses and the ground state of cold dense matter

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Abstract. We study the consequences of recent progress in the experimental determination of masses of neutron rich nuclei for our knowledge of the ground state of cold dense matter. The most recent experimental data determine the ground state of cold dense matter up to $\rho \simeq 10^{11} \text{ g cm}^{-3}$. The composition and the equation of state of the ground state of matter, in this density interval, are calculated.

Key words: dense matter - neutron stars

1. Introduction

Neutron stars are believed to be born as very hot objects, with initial temperature exceeding 10^{11}K . At such temperatures matter can be considered as being in a complete thermodynamic equilibrium (catalyzed matter). This approximation is usually extended to further stages of evolution of neutron stars, where the temperature effect on stellar structure can be neglected. Standard neutron star models assume that stellar matter is in its ground state (cold catalyzed matter) (see, e.g., Shapiro & Teukolsky 1983).

The ground state of matter at the densities and pressures, at which all neutrons are bound in nuclei (i.e. below the neutron drip point) can be described by a model formulated in the classical paper of Baym et al. (1971, hereafter referred to as BPS). An essential input for this model are the ground-state masses of atomic nuclei, present in the lattice sites of the crystal, and immersed in an electron gas. At lowest densities, the relevant nuclei are those, whose ground-state masses are determined with high precision by the laboratory measurements. However, at higher densities the nuclei in the ground state of matter become more and more neutron rich. At the time, when the BPS paper was written, the last experimentally studied nucleus, present in the ground state of dense matter, was ^{84}Se ($Z/A = 0.405$), and the maximum density, at which experimentally studied nucleus was present, was found to be $\rho_{\text{max}}^{\text{exp}} = 8.2 \cdot 10^9 \text{ g cm}^{-3}$.

During last two decades significant progress has been made in the experimental nuclear physics techniques, and masses of many new neutron rich isotopes have been measured (Wapstra & Audi (1985); latest up-to-date results, used in the present work, have been evaluated by Audi (1992, 1993)). As we will show, the present experimental knowledge of the masses of neutron rich atomic nuclei enables determination of the composition and equation of state of the ground state of dense matter up to about $\rho_{\text{max}}^{\text{exp}} \simeq 10^{11} \text{ g cm}^{-3}$.

The model of dense matter is briefly described in Sect. 2. Ground state nuclear masses - available experimentally or evaluated via recent mass formulae are discussed in Sect. 3. Numerical results for the composition and the equation of state of cold catalyzed matter are presented in Sect. 4. We also discuss there possible effects of experimental uncertainties. Sect. 5 contains discussion of our results, with particular emphasis on the comparison between the results based on the experimental data and those based on theoretical calculations. Finally, Sect. 6 presents our conclusions.

2. Model of dense matter

We shall assume that matter is in its ground state (complete thermodynamic equilibrium - cold catalyzed matter) and that it forms a perfect crystal with a single nuclear species, (A, Z) , at lattice sites. Our formalism is similar as in BPS, with some modifications. At given pressure P the equilibrium value of A, Z is determined from the condition that the Gibbs energy per nucleon be minimum. The calculation is done using the Wigner-Seitz (W-S) approximation. The Gibbs energy of the W-S cell is given by

$$G_{cell}(A, Z) = W_N(A, Z) + W_L(Z, n_N) + [\epsilon_e(n_e, Z) + P]/n_N \quad (1)$$

where W_N is the energy of the nucleus (including rest energy of nucleons), W_L is the body-centered cubic lattice energy per cell (BPS), and ϵ_e is the mean electron energy density. The Gibbs energy per nucleon $g = G_{cell}/A$.

There are some differences between our expression for G_{cell} , and the BPS one. Our values of W_N have been obtained from the atomic masses by subtracting not only the electron rest energies, but also removing the atomic electron binding energies. Atomic binding energies were kept in the BPS definition of W_N , to simulate the electron screening

effects in dense matter. In contrast to BPS, we use a much better approximation for the electron screening effects in dense matter. Our expression for ϵ_e takes into account deviations of the electron density from uniformity, which result from the electron screening effects. We include also the exchange term in ϵ_e , which was neglected in BPS. For $\rho \gg 10^6 \text{ g cm}^{-3}$ electrons are ultrarelativistic, and our formula for ϵ_e becomes $\epsilon_e(n_e, Z) = (1.00116 - 1.78 \times 10^{-5} Z^{4/3}) \epsilon_e^{\text{FG}}(n_e)$, where $\epsilon_e^{\text{FG}}(n_e)$ is the energy density of an uniform, free Fermi gas (Salpeter 1961).

At given pressure, the values of electron density, n_e , and number density of nuclei, n_N , are determined from the relations

$$\begin{aligned} n_e &= Z n_N , \\ P &= P_e(n_e, Z) + P_L(n_N, Z) . \end{aligned} \tag{2}$$

At the pressure P_i at which optimal values A, Z change into A', Z' , matter undergoes a baryon density jump, Δn_b , which to a very good approximation is given by the formula

$$\frac{\Delta n_b}{n_b} \cong \frac{Z}{A} \frac{A'}{Z'} - 1 \tag{3}$$

The above equation results from the continuity of pressure, which is to a very good approximation equal to the electron pressure, P_e .

Actually, sharp discontinuity in n_b is a consequence of the assumed one component plasma model. Detailed calculations of the ground state of dense matter by Jog & Smith (1982) show, that the transition between the A, Z and A', Z' shells takes places through a very thin layer of a mixed lattice of these two species. However, since the pressure interval within which the mixed phase exists is typically $\sim 10^{-4} P_i$, the approximation of a sharp density jump is quite a good representation of a nuclear composition of the ground state of matter.

3. Nuclear masses

Experimental masses of nuclei were taken from a recent evaluation of Audi (1992, 1993). Because of the pairing effect, only even-even nuclei are relevant for the ground state problem. In Fig. 1 we show the network of all available experimental masses of even-even nuclei, represented by filled circles. For the remaining isotopes, up to the last one stable with respect to emission of a neutron (proton) pair, we have used theoretical masses obtained with recent mass formula (Möller (1992); the description of the formalism can be found in Möller & Nix (1988)). These nuclides are represented by open circles.

4. Numerical results

4.1. Results obtained using mean experimental masses

The baseline calculation has been done using the (mean) experimental atomic masses, i.e. without considering experimental errors. As it will be discussed in Sect. 4.2, these errors may become substantial for nuclei with largest neutron excess. The equilibrium nuclides present in the cold catalyzed matter are listed in Table 1. In the fifth column we give the maximum density at which a given nuclide is present, ρ_{\max} . In the sixth column we give the value of the electron chemical potential, μ_e , at the density ρ_{\max} . The transition to the next nuclide has a character of a first order phase transition and is accompanied by a density jump. The corresponding fractional increase of density, $\Delta\rho/\rho$, is shown in the last column of Table 1. The last row above the horizontal line, dividing the table into two parts, corresponds to the maximum density, at which the ground state of dense matter is determined by present up-to-date experimental data, ρ_{\max}^{exp} . The table could be extended

to higher densities only by using theoretical determination of nuclear masses. We used the mass predictions of Möller (1992). The very last line of Table 1 corresponds to the neutron drip point in the ground state of dense cold matter. This limiting density can be determined exclusively by the theoretical calculation.

The values of N and Z , corresponding to the ground state of dense matter, are indicated in Fig.1 by large crosses.

The last "experimental" row of Table 1 deserves a comment. The nuclide there is the most neutron rich isotope of nickel, whose mass has been evaluated from experimental data. In view of this, the determination of the ground state of matter had to involve the mass formula for the calculation of the mass of the next nickel isotope, i.e., ^{80}Ni . However, as long as our result for the ground state does not depend on the *theoretical* mass formula used in the vicinity of ^{78}Ni , our result can be treated as reliable. We checked this by switching to another recent theoretical mass formula of Pearson and collaborators (1992; private communication, for the description of the model see Aboussir et al. (1992)); the experimentally known nuclides in the ground state of dense matter did not change. Therefore, the determination of the ground state nuclide in the last row of Table 1 should be treated as based on experiment (as long as we neglect the experimental errors, see below). However, the determination of the value of $\rho_{\text{max}}^{\text{exp}}$ depends on the mass formula used, because the next nuclide in the ground state of dense matter turns out to be ^{126}Ru , sufficiently far from the most neutron rich isotope of Ru experimentally available, ^{116}Ru , that its mass depends in a nonnegligible way on the mass formula used for the extrapolation. In view of this, the pressure (and density) at which the transition from ^{78}Ni to ^{126}Ru takes place depends, to some extent, on the mass formula used to the extrapolation from the experimental nuclear masses (see Tables 1, 2). We get $\rho_{\text{max}}^{\text{exp}} = 9.64 \cdot 10^{10} \text{ g cm}^{-3}$ for the

Möller (1992) mass formula (Table 1), and $\rho_{\text{max}}^{\text{exp}} = 1.08 \cdot 10^{11} \text{ g cm}^{-3}$ for the mass formula of Pearson and collaborators (1992) (Table 2).

The effect of the closed proton and neutron shells on the composition of the ground state of matter is very strong ; except for the ^{56}Fe nucleus, present in the ground state at lowest densities, all nuclides, whose ground state mass is experimentally known, are those with a closed proton or neutron shell (Table 1, Fig. 1). A sequence of three increasingly neutron rich isotopes of nickel $Z = 28$ is followed by a sequence of $N = 50$ isotones of decreasing Z , ending at the last experimentally measured ^{78}Ni . This last nuclide is doubly magic ($N = 50$, $Z = 28$). As we will discuss in the next section, this strong evidence for the effect of closure of the $N = 50$ shell is, at highest densities considered, somewhat weakened when the experimental uncertainties are taken into account.

The equation of state constitutes an essential input for the calculation of the neutron star structure. In Table 3 we give equation of state for the ground state of matter for $\rho < \rho_{\text{max}}^{\text{exp}}$. It has been calculated using the mean experimental masses of atomic nuclei. For the sake of an easier comparison, we used the same pressure grid as that used by BPS (except for the last line, which corresponds to $\rho_{\text{max}}^{\text{exp}}$). Generally, our "experimental" equation of state is very similar to the BPS one. However, in several density intervals one notices a few percent difference, resulting from the difference in the nuclides present at these densities.

At the density exceeding $\rho_{\text{max}}^{\text{exp}}$, and up to the neutron drip density, we get a sequence of $N = 82$ isotones, of decreasing proton number, from $Z = 44$ down to $Z = 36$ for the mass formula of Möller (1992), with neutron drip at $\rho_{\text{ND}} = 4.3 \cdot 10^{11} \text{ g cm}^{-3}$ (Table 1). The results obtained using the theoretical mass formula of Pearson and collaborators (1992) (Table 2) are quite similar to those obtained using the mass formula of Möller (1992), the

differences reducing to two details: the last nucleus before neutron drip point has $Z = 38$ (instead of $Z = 36$ for Möller (1992)), and the neutron drip takes place at a somewhat lower density, $\rho_{\text{ND}} = 4.1 \cdot 10^{11} \text{ g cm}^{-3}$.

More detailed discussion of the importance of the shell terms in various mass formulae, and in particular their comparison with experimental evidence, will be presented in §5.

The ground state composition at given pressure corresponds to the absolute minimum of the Gibbs energy per nucleon, g , in the $N - Z$ plane. Typically, there is only one well distinguished minimum (Fig. 2); only close to the transition pressure between the two nuclear species a well pronounced second minimum appears, and with increasing pressure becomes a new absolute minimum. As seen in Fig. 2, the absolute minimum lies in a valley, which may be called a "beta stability valley" in superdense matter. With increasing pressure, the valley shifts in the N direction, with only a slight change of the inclination angle (Fig. 3).

4.2. Implications of experimental uncertainties

Typical errors in experimentally measured masses of nuclei near the normal (vacuum) beta stability valley are so small (at most a few tens of keV), that they are insignificant for the determination of the value of g : they imply "experimental" uncertainty in g of the order of tens of eV. However, neutron excess in the nuclei present in the ground state of dense matter increases with density. At $\rho = 2 \times 10^9 \text{ g cm}^{-3}$ the minimum of g corresponds to ^{86}Kr ; the experimental mass of this nucleus is known within $(\Delta W)_{\text{exp}} = 5 \text{ keV}$, which corresponds to an insignificant uncertainty $(\Delta g)_{\text{exp}} = 60 \text{ eV}$. At $\rho \simeq 2 \times 10^{10} \text{ g cm}^{-3}$, the ground state nucleus turns out to be ^{82}Ge , with $(\Delta W)_{\text{exp}} = 155 \text{ keV}$, which implies

$(\Delta g)_{\text{exp}} = 2 \text{ keV}$. Still, this uncertainty is not important for the determination of the ground state of matter. However, as one moves to still higher pressure, the experimental uncertainties in W become more important. We found, that of particular importance are large uncertainties in the experimental masses of ^{76}Ni , ^{78}Ni and ^{82}Zn . In what follows, we quote the results of Audi (1993). The latest *mean* experimental atomic mass excesses for ^{76}Ni , ^{78}Ni , ^{80}Zn and ^{82}Zn are, respectively, -42500 keV , -34920 keV , -51780 keV , and -42410 keV . The experimental errors are: $\Delta W(^{82}\text{Zn}) = 400 \text{ keV}$, $\Delta W(^{76}\text{Ni}) = 900 \text{ keV}$, $\Delta W(^{80}\text{Zn}) = 170 \text{ keV}$, and $\Delta W(^{78}\text{Ni}) = 1100 \text{ keV}$. These experimental uncertainties determine upper and lower experimental bounds on W : $W_{\text{u.b.}} = W + \Delta W$, and $W_{\text{l.b.}} = W - \Delta W$. In order to visualize the possible effect of the experimental errors, we performed the calculations of the ground state composition by replacing the masses of ^{76}Ni and ^{82}Zn by their lower bounds, and the masses of ^{78}Ni and ^{80}Zn by the corresponding upper bounds. The resulting modified lines of Table 1 are shown in the upper part of Table 4. Then we repeated the calculations, replacing masses of ^{76}Ni , ^{78}Ni , ^{80}Zn by their upper bounds, and that of ^{82}Zn by its lower bound. Modified lines of Table 1 are displayed in this case in the lower part of Table 4. Both sets of modifications of atomic masses are unfavorable for the appearance of the doubly magic ^{78}Ni in the ground state of dense matter.

Depending of our choice of experimentally allowed masses for the set ^{76}Ni , ^{78}Ni , ^{80}Zn , ^{82}Zn , a specific sequence of increasingly neutron rich nuclides closes the experimental region of the ground state of dense matter.

The results of Table 4 reflect the fact, that the valley in $g(N, Z)$ in the vicinity of g_{min} (Fig. 2, 3) is flat. A specific example of the immediate neighbourhood of the ground state minimum is shown in Fig. 4. Actually, by allowing the uncertainty $\pm(\Delta W)_{\text{exp}}$ we introduce the uncertainty (at a given pressure) in the ground state values of Z and N . Within

experimental uncertainties, the ground state experimental nuclei at $\rho \gtrsim 5 \times 10^{10} \text{ g cm}^{-3}$ are then no longer restricted to those with the $N = 50$ closed shell. For some experimentally allowed combinations of nuclear masses, $N = 48$ or $N = 52$ appear in a density interval (Table 4). For both combinations of atomic masses, the first non-experimental nuclide appears at a somewhat lower density, than in the case when mean experimental atomic masses were used. Also, the first non-experimental $N = 82$ nuclide is that with $Z = 46$, instead of $Z = 44$. This is a consequence of using an upper bound for the atomic mass of ^{78}Ni .

The uncertainty in the "experimental" ground state composition of matter at highest densities implies a corresponding uncertainty in the equation of state. In the density intervals, in which the ground state nuclide is uncertain, the uncertainty in the "experimental" equation of state may reach a few percent.

5. Discussion

In the present paper we calculated the composition and the equation of state of the ground state of dense matter, using the most recent experimental data on masses of very neutron rich nuclei. The experimental data determine the composition of the ground state of dense matter up to $\rho_{\text{max}}^{\text{exp}} \simeq 10^{11} \text{ g cm}^{-3}$. This conclusion does not depend on the particular choice of theoretical determination of nuclear masses via the most recent mass formulae, outside experimentally available region.

If we vary nuclear masses within their experimental bounds, the persistence of the magic number $N = 50$ in the experimentally determined ground state at $\rho \gtrsim 5 \times 10^{10} \text{ g cm}^{-3}$ becomes somewhat weakened. Had we used nuclear masses determined via

theoretical mass formulae (fitted to some earlier experimental data), the strong effect of the closed $N = 50$ shell would leave no possibility for the appearance of the $N = 48, 52$ nuclei in this density interval.

By comparing experimental data for the ^{76}Ni , ^{76}Ni , ^{80}Zn or ^{82}Zn masses with their evaluation via theoretical mass formulae, one notices a systematic overbinding of the $N = 50$ isotopes with respect to their neighbours, and with respect to experiment (see Table 5). The noticeable exception from this trend are results of Pearson and collaborators (1992) and those given by the model of Möller & Nix (1988).

Summarizing, both the calculation of the composition of the ground state of dense cold matter, and the direct comparison with experimental atomic masses indicate, that the additional binding resulting from the closure of the $N = 50$ shell for $Z/A \simeq 0.36 - 0.39$ nuclei is somewhat weaker than that resulting from the shell term in most of the theoretical mass formulae.

While the persistence of the $N = 50$ and/or $Z = 28$ nuclei at lower density may be treated as an *experimental fact*, the strong effect of the $N = 82$, dominating at $\rho_{\text{max}}^{\text{exp}} < \rho < \rho_{\text{ND}}$, might be an artifact of the extrapolation via the semiempirical mass formulae. Some many body calculations of the masses of very neutron rich nuclei suggest, that the effect of the closed $N = 82$ shell might be much weaker, and could be replaced by the strong effect of the closure of the $Z = 40$ subshell (Haensel, Zdunik & Dobaczewski 1989).

6. Conclusion

Using latest evaluation of experimental atomic masses of very neutron rich nuclei, we were able to determine the ground state of cold dense matter for the density up to

$\simeq 10^{11} \text{ g cm}^{-3}$. By varying, within the present day experimental uncertainties, the masses of nuclei in the vicinity of $Z = 28$, $N = 50$, we were able to show, that the effect of the closure of the $N = 50$ shell could be, at $Z/A \simeq 0.36 - 0.39$, somewhat weaker than that obtained from most of existing theoretical mass formulae. We hope, that the future progress in the mass evaluation of the neutron rich nuclei, will eventually yield more precise experimental information about the actual effect of the very large neutron excess on the additional binding due to the closure of neutron shells. The knowledge of this effect turns out to be important for the determination of the ground state of cold, dense matter. This may seem to be of mostly academic interest, because of expected deviations of the real neutron star crust from the ground state. However, this effect might be important also for the detailed astrophysical scenarios of the cosmic nucleosynthesis of heavy nuclei.

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Figure caption

Fig.1. The network of even-even nuclei used in the present calculation. Filled circles - experimental masses; open circles - values extrapolated using mass formula of Möller (1992). The nuclides present in the ground state of matter are indicated by crosses.

Fig.2. Map of $g - g_{min}$ in the $N - Z$ plane, for even - even nuclei, at $P = 1.27 \cdot 10^{27} \text{ dyn cm}^{-2}$. The distance between contour lines is 50 keV. The absolute minimum corresponding to the ground state is indicated by a cross.

Fig.3. Map of $g - g_{min}$ in the $N - Z$ plane, for even - even nuclei, at $P = 4.4 \cdot 10^{29} \text{ dyn cm}^{-2}$.
Notation as in Fig. 2.

Fig.4. The excess of G_{cell} (in keV) with respect to that corresponding to the absolute minimum at $P = 8 \cdot 10^{28} \text{ dyn cm}^{-2}$, in the immediate neighbourhood of the minimum.

Figure 1

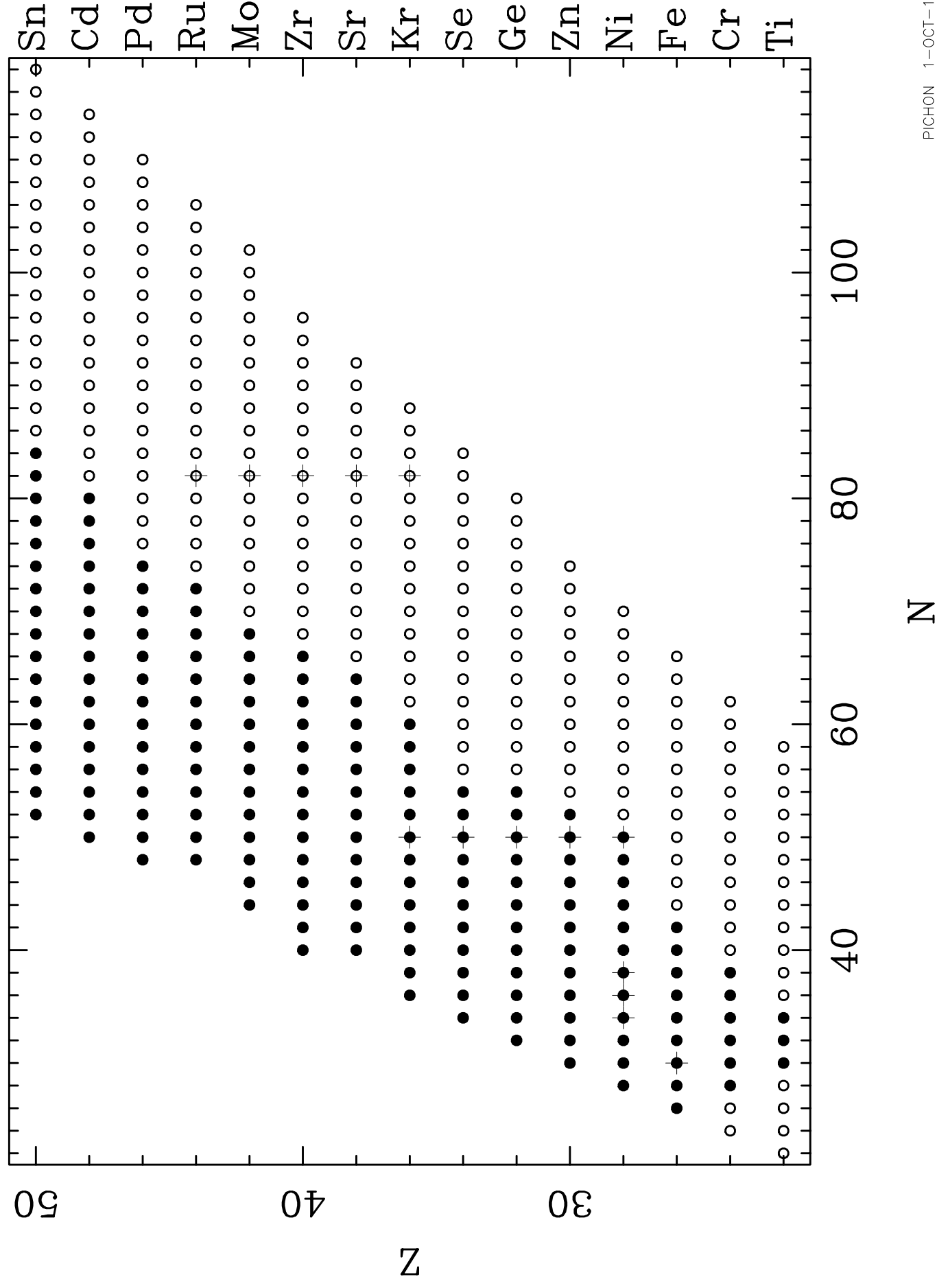


Figure 2 : $P = 1.27000E+27 \text{ dyn cm}^{-2}$

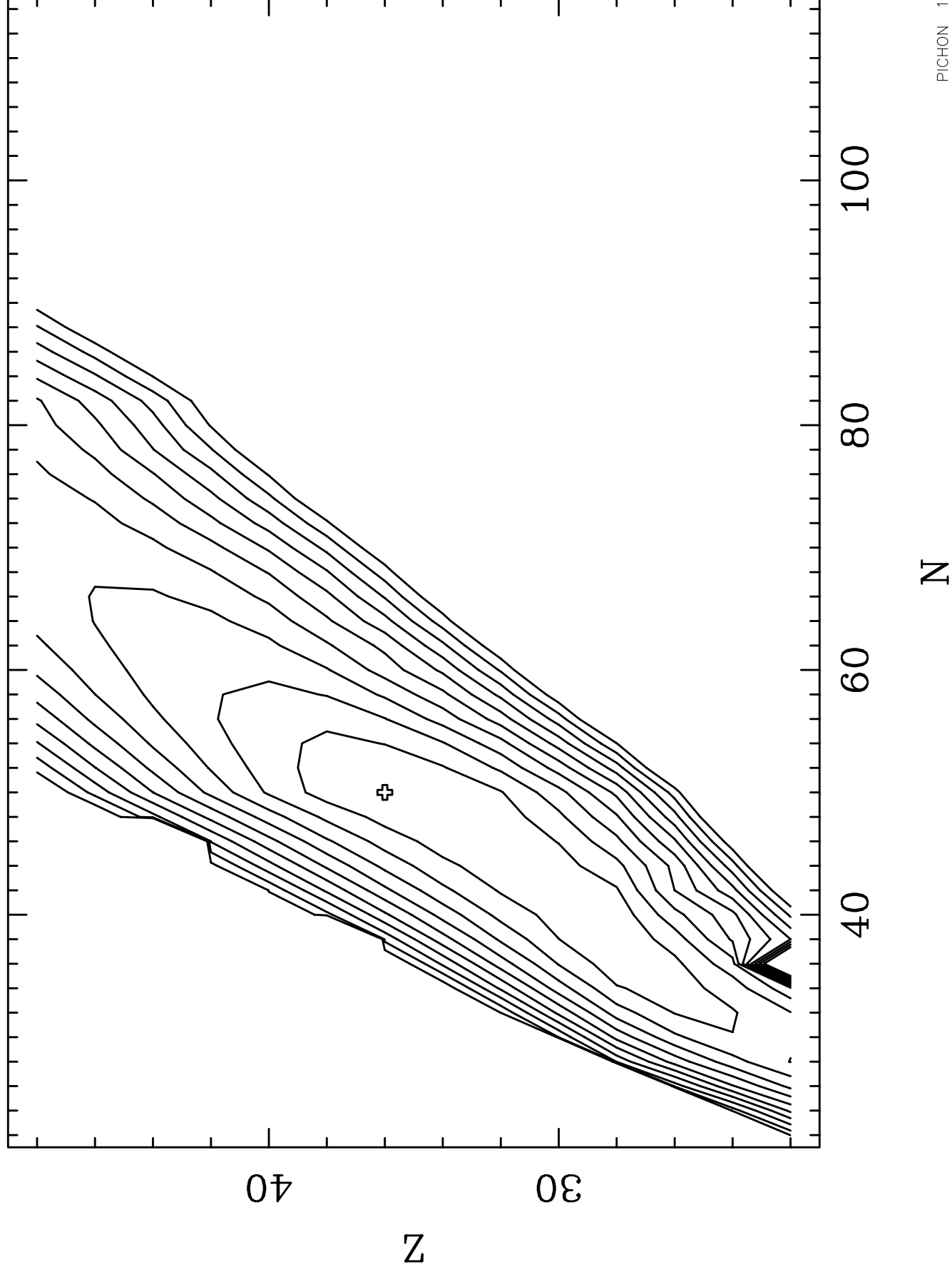


Figure 3 : $P = 4.40000E+29 \text{ dyn cm}^{-2}$

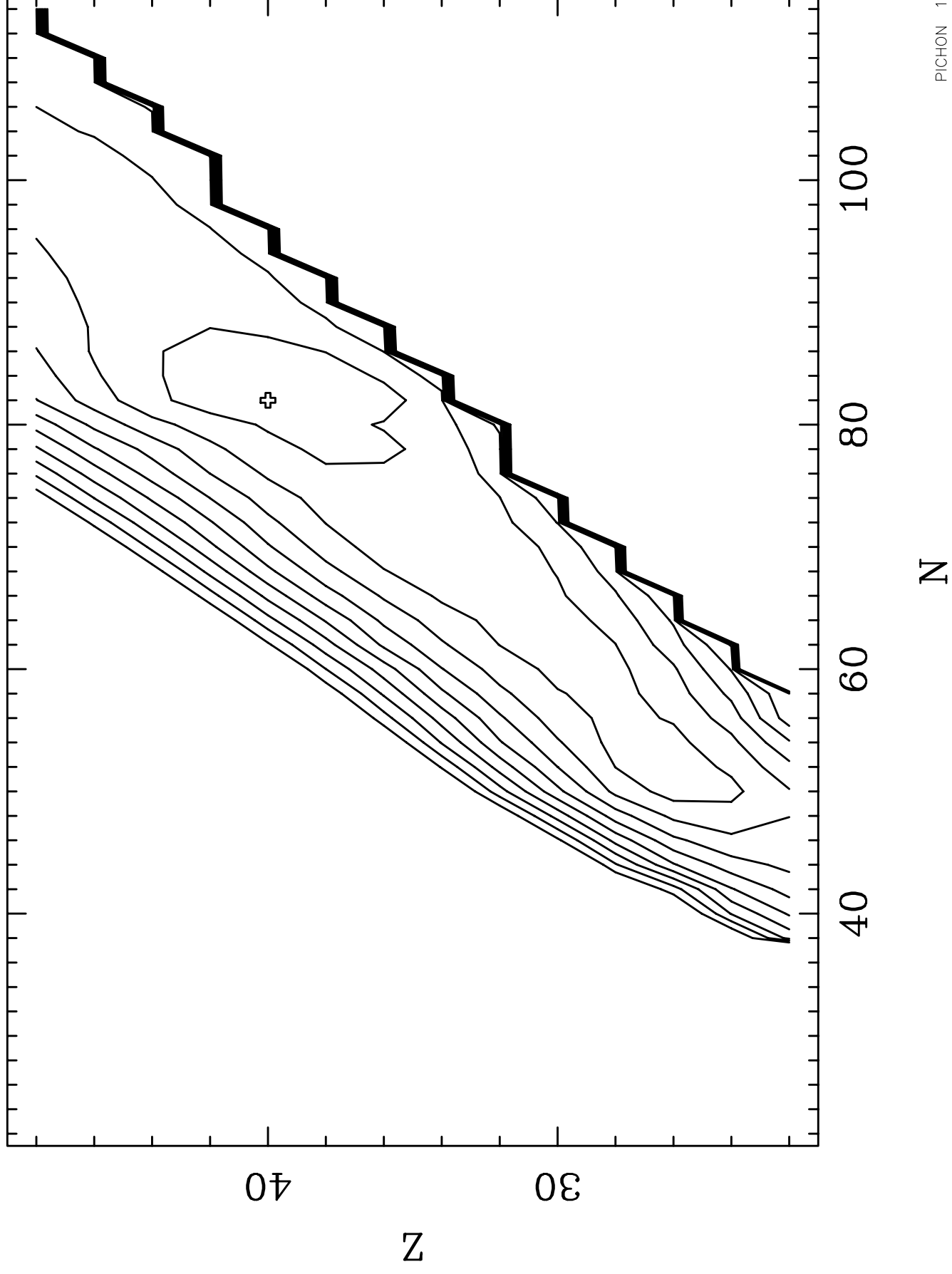


Figure 4 : $P = 8.000000E+28 \text{ dyn cm}^{-2}$

